Translation:Attempt of a Theory of Electrical and Optical Phenomena in Moving Bodies/Section III

1 Investigation of oscillations excited by oscillating ions.

1.1 General formulas.

§ 30. Once the motion of the ions is given, known functions of *x, y, z* and *t* appear on the right-hand side of equations (A) and (B) (§ 21); with respect to the last variable, these are periodic functions if the ions carry out oscillations with constant amplitude and a common oscillation interval *T*. It is easy to see, that in this case the equations are satisfied by values of $\mathfrak{d}_x$, $\mathfrak{d}_y$, $\mathfrak{d}_z$, $\mathfrak{H}_x$, $\mathfrak{H}_y$, $\mathfrak{H}_z$, which also have the period *T*. Therefore, the important and almost self-evident theorem is given:

If ion oscillations of period *T* take place in a light source, then $\mathfrak{d}$ and $\mathfrak{H}$ indicate the same periodicity at each point that shares the translation of the source.

The resolution of the equations leads to quite complicated expressions. For simplicity, it is advisable to calculate the components of the vector $\mathfrak{H}'$ (§ 20) at first.

According to (VI$_\mathrm{b}$)

$$\mathfrak{H}'_x = \mathfrak{H}_x - 4\pi\left(\mathfrak{p}_y\mathfrak{d}_z - \mathfrak{p}_z\mathfrak{d}_y\right).$$

Accordingly, we want to multiply the second and third of equations (A) by $4\pi\mathfrak{p}_z$ and $-4\pi\mathfrak{p}_y$ respectively, and then add them to the first of equations (B). We obtain in this way, under consideration of the importance of $\left(\frac{\partial}{\partial t}\right)_1$ (§ 19),

$$V^2\Delta\mathfrak{H}'_x - \left(\frac{\partial^2\mathfrak{H}'_x}{\partial t^2}\right)_1 = 4\pi V^2\left\{\frac{\partial(\rho\mathfrak{v}_y)}{\partial z} - \frac{\partial(\rho\mathfrak{v}_z)}{\partial y}\right\} +$$
$$+4\pi\mathfrak{p}_z\left\{\frac{\partial(\rho\mathfrak{v}_y)}{\partial t} - \mathfrak{p}_x\frac{\partial(\rho\mathfrak{v}_y)}{\partial x} - \mathfrak{p}_y\frac{\partial(\rho\mathfrak{v}_y)}{\partial y} - \mathfrak{p}_z\frac{\partial(\rho\mathfrak{v}_y)}{\partial z}\right\} -$$
$$-4\pi\mathfrak{p}_y\left\{\frac{\partial(\rho\mathfrak{v}_z)}{\partial t} - \mathfrak{p}_x\frac{\partial(\rho\mathfrak{v}_z)}{\partial x} - \mathfrak{p}_y\frac{\partial(\rho\mathfrak{v}_z)}{\partial y} - \mathfrak{p}_z\frac{\partial(\rho\mathfrak{v}_z)}{\partial z}\right\}.$$

§ 31. In the following calculation, magnitudes of order $\mathfrak{p}^2/V^2$ should be neglected. *First*, we neglect on the right-hand side the terms with *two* factors $\mathfrak{p}_x$, $\mathfrak{p}_y$ or $\mathfrak{p}_z$, since we find a similar term in V^2 ; and we therefore retain only

$$4\pi V^2\left\{\frac{\partial(\rho\mathfrak{v}_y)}{\partial z} - \frac{\partial(\rho\mathfrak{v}_z)}{\partial y}\right\} +$$
$$4\pi\left\{\mathfrak{p}_z\frac{\partial(\rho\mathfrak{v}_y)}{\partial t} - \mathfrak{p}_y\frac{\partial(\rho\mathfrak{v}_z)}{\partial t}\right\}.$$

Second, we write for the operation that has to be applied to $\mathfrak{H}'_x$,

$$V^2\Delta - \left(\frac{\partial}{\partial t} - \mathfrak{p}_x\frac{\partial}{\partial x} - \mathfrak{p}_y\frac{\partial}{\partial y} - \mathfrak{p}_z\frac{\partial}{\partial z}\right)^2 =$$
$$\left(V^2\frac{\partial^2}{\partial x^2} + 2\mathfrak{p}_x\frac{\partial^2}{\partial x\,\partial t}\right) +$$
$$+\left(V^2\frac{\partial^2}{\partial y^2} + 2\mathfrak{p}_y\frac{\partial^2}{\partial y\,\partial t}\right) + \left(V^2\frac{\partial^2}{\partial z^2} + 2\mathfrak{p}_x\frac{\partial^2}{\partial z\,\partial t}\right) -$$
$$\frac{\partial^2}{\partial t^2} =$$
$$= V^2\left(\frac{\partial}{\partial x} + \frac{\mathfrak{p}_x}{V^2}\frac{\partial}{\partial t}\right)^2 + V^2\left(\frac{\partial}{\partial y} + \frac{\mathfrak{p}_y}{V^2}\frac{\partial}{\partial t}\right)^2 +$$
$$+V^2\left(\frac{\partial}{\partial z} + \frac{\mathfrak{p}_z}{V^2}\frac{\partial}{\partial t}\right)^2 - \frac{\partial^2}{\partial t^2}.$$

The form of this expression suggests the introduction of a new independent variable instead of *t*

and to consider $\mathfrak{H}'_x$, as well as $\rho\mathfrak{v}_y$ and $\rho\mathfrak{v}_z$, as functions of *x, y, z* and t' . We denote the derivative that corresponds to this view by

$$\left(\frac{\partial}{\partial x}\right)', \left(\frac{\partial}{\partial y}\right)', \left(\frac{\partial}{\partial z}\right)' \text{ and } \frac{\partial}{\partial t'}$$

and give to the sign Δ' the meaning

$$\left(\frac{\partial^2}{\partial x^2}\right)' + \left(\frac{\partial^2}{\partial y^2}\right)' + \left(\frac{\partial^2}{\partial z^2}\right)'$$

It is now

and

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t'},$$

so that we find for the determination of $\mathfrak{H}'$

$$V^2\Delta'\mathfrak{H}'_x - \frac{\partial^2\mathfrak{H}'_x}{\partial t'^2} =$$
$$4\pi V^2\left[\left\{\frac{\partial(\rho\mathfrak{v}_y)}{\partial z}\right\}' - \left\{\frac{\partial(\rho\mathfrak{v}_z)}{\partial y}\right\}'\right] \text{ etc.},$$

A solution of these equations is easy to give. Namely, imagine three functions ψ_x , ψ_y , ψ_z that satisfy the conditions

and put

Once $\mathfrak{H}'$ is found by that, equation (III_b) provides us with the value of $\dot{\mathfrak{d}}$ and thus also, as far as we don't use additive constants, the value of $\mathfrak{d}$. From (VI_b) it also follows $\mathfrak{H}$; from (V_b) and (VII_b) it follows $\mathfrak{F}$ and $\mathfrak{E}$. That in this way really *all* the equations are satisfied, can be proven, but should not be discussed here for brevity.

In contrast, in the next section the value of ψ_x shall be given, and in § 33 the solution for a special case shall be further developed.

It should also be remarked before, that the variable t' can be regarded as a time, counting from an instant that depends on the location of the point. We can therefore call this variable the *local time* of this point, in contrast to the *general time t*. The transition from one time to another is provided by equation (34).

§ 32. The product $\rho\mathfrak{v}_x$ in the first of equations (36), as noted already, is a known function of *x, y, z* and t' . We accordingly set

$$\rho\mathfrak{v}_x = f(x, y, z, t')$$

and thus have

a solution of (36)[1]. By that we have to imagine two points; *first*, the *fixed* point (*x, y, z*), for which we want to calculate ψ_x and which we call *P*; *second*, a *moving* point *Q*, which has to traverse the whole space, where $\rho\mathfrak{v}_x$ is different from zero. *r* represents the distance *QP*, and t' the local time of *P* at the instant for which we wish to calculate ψ_x ; furthermore we have to understand by ξ, η, ζ, the coordinates of *Q*, and by $d\tau$ an element of the just mentioned space. The function $f\left(\xi, \eta, \zeta, t' - \frac{r}{V}\right)$ is the value of $\rho\mathfrak{v}_x$ in this element, namely, if the local time that is valid at this place, is $t' - \frac{r}{V}$.

1.2 A single luminous molecule.

§ 33. To excite electric oscillations, a single molecule with oscillating ions shall serve; let Q_0 be an arbitrary fixed point in it — for brevity, we say, "the molecule is present in Q_0 " —, and for *P* a place is chosen, whose distance from Q_0 is much larger than the dimensions of the molecules. For distinction, $Q_0 P = r_0$.

We now want to replace the various distances *r*, that are present in formula (38), by r_0 and also neglect the differences of local times at the various points of the molecule. In this way,

$$\psi_x = -\frac{1}{r_0}\int \rho\mathfrak{v}_x d\tau,$$

where all occurring $\rho\mathfrak{v}_x$ are related to the *same* instant, namely to the instant when

$$t' = \frac{r_0}{V}$$

is the local time of Q_0 .

Since $\mathfrak{v}_x$ is equal for all points of an ion, then, if we write *e* for the charge of such a particle, the last integral transforms into

$$\Sigma e\mathfrak{v}_x.$$

The sum is extending over all ions of the molecule.

Furthermore, if $\mathfrak{q}$ is now the displacement of an ion from its equilibrium position, then

$$\mathfrak{v}_x = \frac{d\mathfrak{q}_x}{dt},$$

and

$$\Sigma e\mathfrak{v}_x = \frac{d}{dt}\Sigma e\mathfrak{q}_x.$$

This has a simple meaning. We can conveniently call the vector $\Sigma e\mathfrak{q}$ the *electric moment* of the molecule and denote it by $\mathfrak{m}$. Then it is

$$\Sigma e\mathfrak{q}_x = \mathfrak{m}_x, \psi_x = -\frac{1}{r_0}\frac{d\mathfrak{m}_x}{dt} = -\frac{\partial}{\partial t}\left(\frac{\mathfrak{m}_x}{r_0}\right);$$

after the things said here, we have to take the value of the derivative for the instant when the local time in Q_0 is $t' = \frac{r_0}{V}$. Obviously we can also write

$$\psi_x = -\frac{\partial}{\partial t'}\left(\frac{\mathfrak{m}_x}{r_0}\right),$$

where $\mathfrak{m}_x$ means the first component of the electric moment in that very instant. After (by that and by two equations of the same from) we have found ψ_x , ψ_y , ψ_z for the point (*x, y, z*) and the local time t' at this place, the study of the propagating oscillations is very simple. The equations (37) give

and because we seek the value of $\mathfrak{d}$ outside the molecule, (III_b) is transformed into

$$4\pi\dot{\mathfrak{d}} = Rot\ \mathfrak{H}',$$

or, due to (35), it is transformed into

$$4\pi\dot{\mathfrak{d}}_x = \left(\frac{\partial\mathfrak{H}'_z}{\partial y}\right)' - \left(\frac{\partial\mathfrak{H}'_y}{\partial z}\right)' - \frac{\mathfrak{p}_y}{V^2}\frac{\partial\mathfrak{H}'_z}{\partial t} + \frac{\mathfrak{p}_z}{V^2}\frac{\partial\mathfrak{H}'_y}{\partial t}\text{etc.},$$

If we bring the last two terms on the left side, then we just obtain $\frac{1}{V^2}\dot{\mathfrak{F}}_x$ or $\frac{1}{V^2}\frac{\partial\mathfrak{F}_x}{\partial t'}$, as it can be seen by (V_b); since $\mathfrak{H}$ and $\mathfrak{H}'$ only differ by magnitudes of order $\mathfrak{p}$, we may replace the vector product (V_b) by $[\mathfrak{p}.\mathfrak{H}']$.

From

$$\frac{\partial\mathfrak{F}_x}{\partial t'} = V^2\left[\left(\frac{\partial\mathfrak{H}'_z}{\partial y}\right)' - \left(\frac{\partial\mathfrak{H}'_y}{\partial z}\right)'\right]\text{etc.},$$

we obtain $\mathfrak{F}$ by integration; constants were omitted by us, since we are only dealing with vibrations.

We substitute the values (39) and put

$$\left(\frac{\partial}{\partial x}\right)'\left(\frac{\mathfrak{m}_x}{r_0}\right)+\left(\frac{\partial}{\partial y}\right)'\left(\frac{\mathfrak{m}_y}{r_0}\right)+\left(\frac{\partial}{\partial z}\right)'\left(\frac{\mathfrak{m}_z}{r_0}\right)=S.$$

It is then

and namely, $\mathfrak{m}_x$, $\mathfrak{m}_y$, $\mathfrak{m}_z$ are still related to the instant given above.

As to how the other magnitudes occurring in (I_b)-(VII_b) can be determined, can immediately be seen.

§ 34. Just some words on the error committed in the above calculation. That in (38) the factor $\frac{1}{r}$ was replaced by $\frac{1}{r_0}$, needs surely no justification. But we also haven't taken the values of $\rho\mathfrak{v}$ for the function f at the the correct times. Once we have replaced $t'-\frac{r}{V}$ by $t'-\frac{r_0}{V}$ in (38), then in the time when *l* is one of the dimensions of the molecule, we have committed an error of order $\frac{l}{V}$, secondly, the inequality of the local times at the various locations of the molecule were not considered, and in this lies an error of order $\frac{l\mathfrak{p}}{V^2}$ by (34). But even then, if we want to keep magnitudes of the order $\frac{\mathfrak{p}}{V}$, we don't need to care about this second error, when already the first may be neglected. Now this is indeed the case when the dimensions of the molecule are much smaller than the wavelength of *TV*. Then also *l/V* is considerably smaller than *T*, and the state in the molecule will not noticeably change in the time *l/V*.

§ 35. The formulas for the propagation of *oscillations* is obtained, if goniometric functions of time are substituted into the equations (39) and (40) for $\mathfrak{m}_x$, $\mathfrak{m}_y$, $\mathfrak{m}_z$. If, for example,

$$\mathfrak{m}_y=0,\ \mathfrak{m}_z=0,$$

and, as a function of local time which is valid for the location of the molecule,

$$\mathfrak{m}_x=a\cos 2\pi\frac{t'}{T}\text{constant), (a ,}$$

thus at an external point in the distance *r* and for the local time t' that belongs to it

$$\mathfrak{H}'_x=0,\ \mathfrak{H}'_y=\frac{\partial}{\partial t'}\left(\frac{\partial\chi}{\partial z}\right)',\ \mathfrak{H}'_z=-\frac{\partial}{\partial t'}\left(\frac{\partial\chi}{\partial y}\right)',$$

$$\mathfrak{F}_z=-V^2\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)'\chi,\ \mathfrak{F}_y=V^2\left(\frac{\partial^2\chi}{\partial x\,\partial y}\right)',\ \mathfrak{F}=V^2\left(\frac{\partial^2\chi}{\partial x\,\partial z}\right)'.$$

$$\chi=\frac{a}{r}\cos\frac{2\pi}{T}\left(t'-\frac{r}{V}\right).$$

If we eventually want to consider a stationary light source once, so we simply have to omit all accents. The formulas then are in accordance with the expressions, by which Hertz[2] represented the oscillations in the vicinity of his vibrator.

1.3 The direction of the wave normal.

§ 36. Now we shall examine the oscillations in such distances from the luminous molecules, which are considerably larger than the wavelength. It should be noted that in (39) and (40), $\mathfrak{m}_x$, $\mathfrak{m}_y$, $\mathfrak{m}_z$ are goniometric functions of

$$t'-\frac{r}{V}\ ;$$

we namely want to write from now on *r* instead of r_0 . The assumption made about the length of this line justifies to consider only the variability of the argument of any goniometric function for all differentiations with respect to x, y, z, but to consider as constant all factors such as $\frac{1}{r}$, or $\cos(r,x)$, by which these functions are multiplied.

For any of the magnitudes $\mathfrak{H}'_x$, $\mathfrak{H}'_y$, $\mathfrak{H}'_z$, $\mathfrak{F}_x$, $\mathfrak{F}_y$, $\mathfrak{F}_z$ - we will call them φ — it can therefore be found an expression of the form

where *A* and *B* are indeed dependent on the length and the direction of line Q_0P — Q_0 is the location of the molecule, and *P* is the considered external point —, but, if *r* were just big enough, it may be regarded as constant in a space that comprises many wavelengths. While *x, y, z* are the coordinates of *P*, we denote by ξ, η, ζ the coordinates of Q_0 , and by b_x , b_y , b_z the direction constants of the connection-line Q_0P . If we now replace in the formula (41) *r* by

$$b_x(x-\xi)+b_y(y-\eta)+b_z(z-\zeta),$$

and t' by the value (34), we obtain

$$C=B+\frac{1}{V}(b_x\xi+b_y\eta+b_z\zeta).$$

In an area that isn't too extended, we may also regard b_x , b_y , b_z as constant, and thus regard the motion as a system of plane waves. The direction constants b'_x , b'_y , b'_z of the wave normal are obviously to be determined from the condition

For $\mathfrak{p}=0$, b'_x , b'_y , b'_z fall into b_x , b_y , b_z , and the waves are perpendicular to Q_0P . This is not the case if the light source is moving. From (43) follows, that the waves are perpendicular to the line that connects *P* with that point at which the light source was at the moment, when the light was sent that reaches *P* at time *t*.

1.4 The law of Doppler.

§ 37. In a point that moves together with the luminous molecule — and thus also for an observer who shares the translation — the values of $\mathfrak{d}_x$, ...$\mathfrak{H}_x$, ... are changing, as we have seen (§ 30), as often in unit time as it corresponds to the actual period of oscillation *T* of the ions.

We can also examine, with which frequency these values in a *stationary* point are changing their sign. This frequency causes *the oscillation period for a stationary observer*. The question can be solved immediately, if instead of *x, y, z* we introduce new coordinates **x** , **y** , **z** , which refer to a *stationary* system of axes. If the two systems have the same directions of axes and the same origin at time *t* = 0, then

and by (42) for $\mathfrak{d}_x, \ldots \mathfrak{H}_x, \ldots$ we obtain expressions of the form

$$A \cos \tfrac{2\pi}{T} \left\{ t + \tfrac{\mathfrak{p}_r}{V} t - \left(\tfrac{b_x}{V} + \tfrac{\mathfrak{p}_x}{V^2} \right) \mathbf{x} - \text{etc.} \ldots + C \right\},$$

where

$$\mathfrak{p}_r = b_x \mathfrak{p}_x + b_y \mathfrak{p}_y + b_z \mathfrak{p}_z$$

is the component of $\mathfrak{p}$ with respect to the connection line $Q_0 P$.

The "observed" period of oscillation is thus

$$T' = \tfrac{T}{1 + \frac{\mathfrak{p}_r}{V}} = T \left(1 - \tfrac{\mathfrak{p}_r}{V} \right),$$

what is in agreement with the known law of Doppler[3]. If the law, as it is usually applied, should be given, it must of course still be assumed, *that the translation does not change the actual period of oscillation of the luminous particles*. I must abstain from giving an account of this hypothesis, since we know nothing about nature of the molecular forces that determine the oscillation period.

§ 38. The case that the light source is at rest and the observer progresses, allows of a similar treatment. If namely, as above, **x** , **y** , **z** are the coordinates based on stationary axes, then in a distant point *P*, any of the magnitudes $\mathfrak{d}_x, \ldots \mathfrak{H}_x, \ldots$ shall now be represented by

We most conveniently describe the perception of motion by means of a co-ordinate system, which shares the translation $\mathfrak{p}$ of the observer. Here, again the relations (44) are applicable, and (46) transforms into

$$A \cos \tfrac{2\pi}{T} \left\{ t - \tfrac{\mathfrak{p}_r}{V} t - \tfrac{b_x x + b_y y + b_z z}{V} + C \right\},$$

from which it is given for the "observed" period of oscillation

$$T' = \tfrac{T}{1 - \frac{\mathfrak{p}_r}{V}} = T \left(1 + \tfrac{\mathfrak{p}_r}{V} \right).$$

[1] The proof for this can be found, for example, in my treatise: La théorie électromagnétique de Maxwell et son application aux corps mouvants.

[2] Hertz. Wied. Ann., Bd. 36, p. 1, 1889.

[3] The derivation given here can easily be generalized so that it can be applied to all similar cases, for example also to sounding bodies. An arbitrary body *A* move with constant velocity $\mathfrak{p}$ in a medium that either remains at rest, or comes into a *stationary* state of motion. In this latter case (which also encloses the former one) we find at any point *P*, *which translates with the body A*, always the same state of motion, and it can be said, that the whole figure representing the distribution of velocities in the vicinity of *A*, shares the translation $\mathfrak{p}$.
Furthermore, imagine now that the parts of the body perform simple oscillations of period *T* and of constant amplitude. It seems clear without further ado, when a sufficiently long time has elapsed since the beginning of this motion, that in the just-mentioned point *P*, the deviation from equilibrium or rather from the stationary state of flow, must necessarily have the period *T*. If we now introduce the co-ordinates *x, y, z* with respect to a system of axes progressing with the body (relative coordinates), and if we restrict ourselves to a space, that is so far from *A* and so small that we can speak of plane waves in it, then the above deviation can be represented by expressions of the form Here, a_x , a_y , a_z are the direction constants of the wave normal, while *V* is the velocity of propagation. If we now want to know, by which frequency φ (in a *stationary* point) its sign is varying, then we have to introduce coordinates **x** , **y** , **z** with respect to stationary axes. By using the relations (44), (45) transforms into

$$\phi = A \cos \tfrac{2\pi}{T} \left\{ t + \tfrac{\mathfrak{p}_n}{V} t - \tfrac{a_x \mathbf{x} + a_y \mathbf{y} + a_z \mathbf{z}}{V} + p \right\},$$

where

$$\mathfrak{p}_n = a_x \mathfrak{p}_x + a_y \mathfrak{p}_y + a_z \mathfrak{p}_z$$

are the components of $\mathfrak{p}$ with respect to the wave normal For the observed oscillation period we now obtain

$$T' = \tfrac{T}{1 + \frac{\mathfrak{p}_n}{V}} = T \left(1 - \tfrac{\mathfrak{p}_n}{V} \right).$$

What we have already stated without proof, namely that the period *T* exists throughout in the medium, is nothing else than what Petzval, in his attacks against Doppler's theory, called the law of the immutability of the oscillation period (Wiener Sitz.-Ber., vol 8, p. 134, 1852). He only forgot to notice, that this law only would apply, if we consider the phenomena as a function of *t* and the *relative* coordinates. The proof of the theorem is, by the way, easy to give, when the oscillations are infinitely small, and when we have to do with homogeneous linear differential equations. As regards the acoustic phenomena, the problem was discussed in detail by Was (Het beginsel van Doppler in de geluidsleer, Leiden, Engels, 1881).

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2.1 Text

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